3.5 Free Fall

Learning Objectives

By the end of this section, you will be able to:

- Use the kinematic equations with the variables y and g to analyze free-fall motion.
- Describe how the values of the position, velocity, and acceleration change during a free fall.
- Solve for the position, velocity, and acceleration as functions of time when an object is in a free fall.

An interesting application of through is called *free fall*, which describes the motion of an object falling in a gravitational field, such as near the surface of Earth or other celestial objects of planetary size. Let's assume the body is falling in a straight line perpendicular to the surface, so its motion is one-dimensional. For example, we can estimate the depth of a vertical mine shaft by dropping a rock into it and listening for the rock to hit the bottom. But "falling," in the context of free fall, does not necessarily imply the body is moving from a greater height to a lesser height. If a ball is thrown upward, the equations of free fall apply equally to its ascent as well as its descent.

Gravity

The most remarkable and unexpected fact about falling objects is that if air resistance and friction are negligible, then in a given location all objects fall toward the center of Earth with the *same constant acceleration*, *independent of their mass*. This experimentally determined fact is unexpected because we are so accustomed to the effects of air resistance and friction that we expect light objects to fall slower than heavy ones. Until Galileo Galilei (1564–1642) proved otherwise, people believed that a heavier object has a greater acceleration in a free fall. We now know this is not the case. In the absence of air resistance, heavy objects arrive at the ground at the same time as lighter objects when dropped from the same height Figure 3.29.

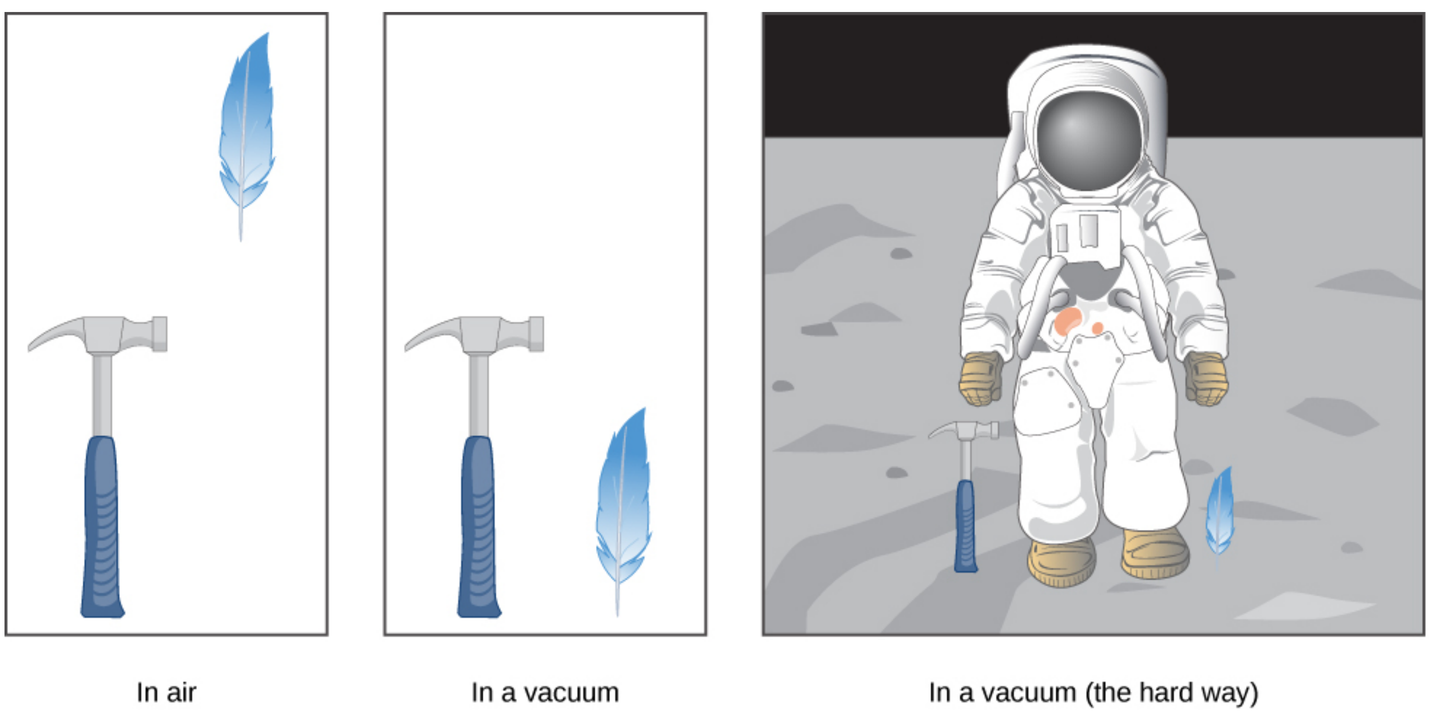

Figure 3.29 A hammer and a feather fall with the same constant acceleration if air resistance is negligible. This is a general characteristic of gravity not unique to Earth, as astronaut David R. Scott demonstrated in 1971 on the Moon, where the acceleration from gravity is only 1.67 m/s^2 and there is no atmosphere.

In the real world, air resistance can cause a lighter object to fall slower than a heavier object of the same size. A tennis ball reaches the ground after a baseball dropped at the same time. (It might be difficult to observe the difference if the height is not large.) Air resistance opposes the motion of an object through the air, and friction between objects—such as between clothes and a laundry chute or between a stone and a pool into which it is dropped—also opposes motion between them.

For the ideal situations of these first few chapters, an object *falling without air resistance or friction* is defined to be in **free fall**. The force of gravity causes objects to fall toward the center of Earth. The acceleration of free-falling objects is therefore called **acceleration due to gravity**. Acceleration due to gravity is constant, which means we can apply the kinematic equations to any falling object where air resistance and friction are negligible. This opens to us a broad class of interesting situations.

Acceleration due to gravity is so important that its magnitude is given its own symbol, g. It is constant at any given location on Earth and has the average value

$$g = 9.81\ \text{m/s}^2\ \ (\text{or } 32.2\ \text{ft/s}^2)$$

Although g varies from 9.78 m/s^2 to 9.83 m/s^2, depending on latitude, altitude, underlying geological formations, and local topography, let's use an average value of 9.8 m/s^2 rounded to two significant figures in this text unless specified otherwise. Neglecting these effects on the value of g as a result of position on Earth's surface, as well as effects resulting from Earth's rotation, we take the direction of acceleration due to gravity to be downward (toward the center of Earth). In fact, its direction *defines* what we call vertical. Note that whether acceleration a in the kinematic equations has the value $+g$ or$-g$ depends on how we define our coordinate system. If we define the upward direction as positive, then $a = -g = -9.8\ \text{m/s}^2$, and if we define the downward direction as positive, then $a = g = 9.8\ \text{m/s}^2$.

One-Dimensional Motion Involving Gravity

The best way to see the basic features of motion involving gravity is to start with the simplest situations and then progress toward more complex ones. So, we start by considering straight up-and-down motion with no air resistance or friction. These assumptions mean the velocity (if there is any) is vertical. If an object is dropped, we know the initial velocity is zero when in free fall. When the object has left contact with whatever held or threw it, the object is in free fall. When the object is thrown, it has the same initial speed in free fall as it did before it was released. When the object comes in contact with the ground or any other object, it is no longer in free fall and its acceleration of g is no longer valid. Under these circumstances, the motion is one-dimensional and has constant acceleration of magnitude g. We represent vertical displacement with the symbol y.

KINEMATIC EQUATIONS FOR OBJECTS IN FREE FALL

We assume here that acceleration equals – *g* (with the positive direction upward).

$$v = v_0 - gt \tag{3.37}$$

$$y = y_0 + v_0 t - \frac{1}{2}gt^2 \tag{3.38}$$

$$v^2 = v_0^2 - 2g(y - y_0) \tag{3.39}$$

Problem-Solving Strategy

Free Fall

1. Decide on the sign of the acceleration of gravity. In Equation 3.37 through Equation 3.39, acceleration *g* is negative, which says the positive direction is upward and the negative direction is downward. In some problems, it may be useful to have acceleration *g* as positive, indicating the positive direction is downward.
2. Draw a sketch of the problem. This helps visualize the physics involved.
3. Record the knowns and unknowns from the problem description. This helps devise a strategy for selecting the appropriate equations to solve the problem.
4. Decide which of Equation 3.37 through Equation 3.39 are to be used to solve for the unknowns.

Example 3.15

Free Fall of a Ball

Figure 3.30 shows the positions of a ball, at 1-s intervals, with an initial velocity of 4.9 m/s downward, that is thrown from the top of a 98-m-high building. (a) How much time elapses before the ball reaches the ground? (b) What is the velocity when it arrives at the ground?

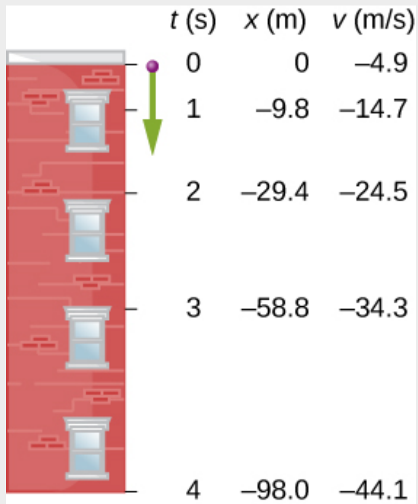

Figure 3.30 The positions and velocities at 1-s intervals of a ball thrown downward from a tall building at 4.9 m/s.

Strategy

Choose the origin at the top of the building with the positive direction upward and the negative direction downward. To find the time when the position is –98 m, we use Equation 3.38, with $y_0 = 0, v_0 = -4.9 \text{ m/s}, \text{and } g = 9.8 \text{ m/s}^2$.

Solution

a. Substitute the given values into the equation:

$$y = y_0 + v_0 t - \frac{1}{2}gt^2$$
$$-98.0 \text{ m} = 0 - (4.9 \text{ m/s})t - \frac{1}{2}(9.8 \text{ m/s}^2)t^2.$$

This simplifies to

$$t^2 + t - 20 = 0.$$

This is a quadratic equation with roots $t = -5.0\ \text{s}$ and $t = 4.0\ \text{s}$. The positive root is the one we are interested in, since time $t = 0$ is the time when the ball is released at the top of the building. (The time $t = -5.0\ \text{s}$ represents the fact that a ball thrown upward from the ground would have been in the air for 5.0 s when it passed by the top of the building moving downward at 4.9 m/s.)

b. Using Equation 3.37, we have

$$v = v_0 - gt = -4.9\ \text{m/s} - (9.8\ \text{m/s}^2)(4.0\ \text{s}) = -44.1\ \text{m/s}.$$

Significance

For situations when two roots are obtained from a quadratic equation in the time variable, we must look at the physical significance of both roots to determine which is correct. Since $t = 0$ corresponds to the time when the ball was released, the negative root would correspond to a time before the ball was released, which is not physically meaningful. When the ball hits the ground, its velocity is not immediately zero, but as soon as the ball interacts with the ground, its acceleration is not *g* and it accelerates with a different value over a short time to zero velocity. This problem shows how important it is to establish the correct coordinate system and to keep the signs of *g* in the kinematic equations consistent.